Chapter Ten

Volumes and Surface Areas of Simple Solids

We look at how to calculate the areas and perimeters of many geometric figures in the last chapter. As you get farther advanced in math, you'll find cases where you need to find the volume of a solid object or the total surface area. Because this is a beginner's book, we're going to cover only a few of the simpler solid objects.

The color illustrations will help you to visualize each problem. Volume is expressed in cubic units. So, for instance, if you have an object measured in inches, the volume would be given in cubic inches. For an object measured in centimeters, the volume would be in cubic centimeters.

Volume and Surface Area of a Rectangular Prism

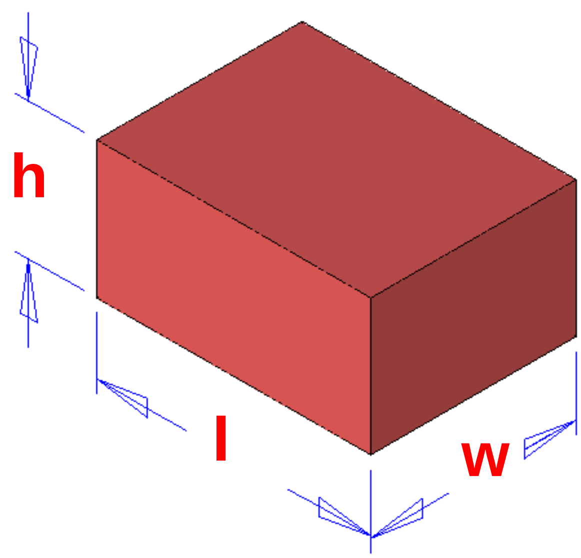
The volume of a rectangular prism is the product of the length, width, and height. This is one of the most common volume calculations that you will do.

For example, this would tell you how much space there is in a room of your house or how much water an aquarium holds.

Volume = length x width x height

Volume = l x w x h

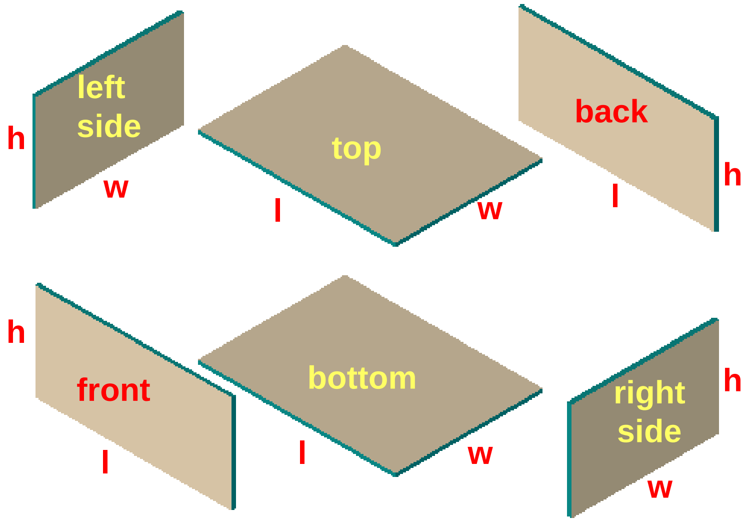
Volume = lwh



The surface area of a rectangular prism is also easy to calculate.

First, you will notice that for any given rectangular prism, there are three sets of faces.

The top and bottom face are the same, the front and backfaces are the same, and the left and right faces are the same.



I've shown the faces of the rectangular prism in exploded view to the left to simplify our discussion.

The surface area of the rectangular prism is the sum of the areas of all six faces.

Here are the area calculations for the six faces.

Area of front = l x h
Area of back = l x h

Area of top = l x w
Area of bottom = l x w

Area of left side = w x h
Area of right side = w x h

So the total surface area can be calculated as follows:

Total Area = 2 (l x h) + 2 (l x w) + 2 (w x h)

Let's solve a practical problem using what we have just learned. Suppose that you need a storage space that has at least 1200 cubic feet for a project. Your friend offers the use of a room that is 12ft wide x 14ft long x 8 ft high. Is the room large enough?

You calculate the volume using:

V = l x w x h

V = 14 x 12 x 8 = 1344 cubic feet.

So the room is large enough to use.

Let's say that you decide to paint the four walls and the ceiling prior to putting your stuff in the room. When you look at paint at the store, the label says that a gallon will cover 300 square feet. How many gallons of paint should you buy to put one coat on each wall and the ceiling?

The total surface area of the room is calculated using:

$$\text{Total Area} = 2\,(l \times h) + 2\,(l \times w) + 2\,(w \times h)$$

$$\text{Area of floor} = l \times w$$

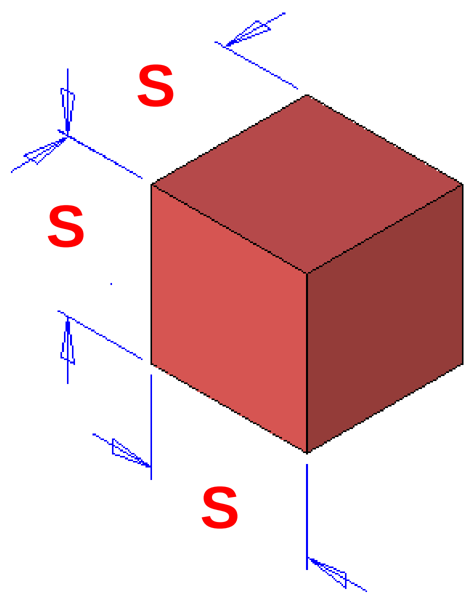
$$\text{Paintable Area} = 2\,(l \times h) + (l \times w) + 2\,(w \times h)$$

$$\text{Paintable Area} = 2\,(14 \times 8) + (14 \times 12) + 2\,(12 \times 8)$$

$$\text{Paintable Area} = 224 + 168 + 192 = 584$$

So it looks like you better buy two gallons.

Volume and Surface Area of a Cube



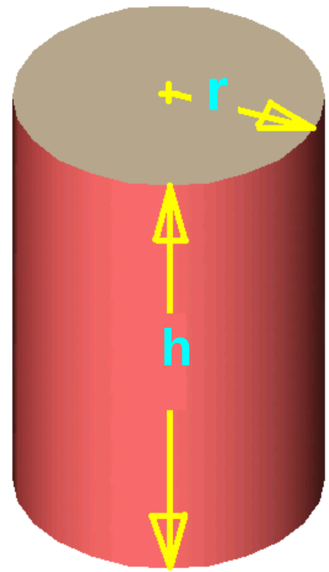
A cube is a special case of a rectangular prism. It has six identical faces. So the length equals the width equals the height.

This makes the calculation of volume and surface area quite easy.

$$\text{Volume} = S \times S \times S = S^3$$

$$\text{Surface Area} = 6 \times S^2$$

Volume and Surface Area of a Cylinder



Calculating the volume and surface area of a cylinder is easy if you use the information that we learned about circles.

The volume of the cylinder is equal to the area of the circular end times the height.

You'll remember that to find the area of a circle we use the following relationship (equation.)

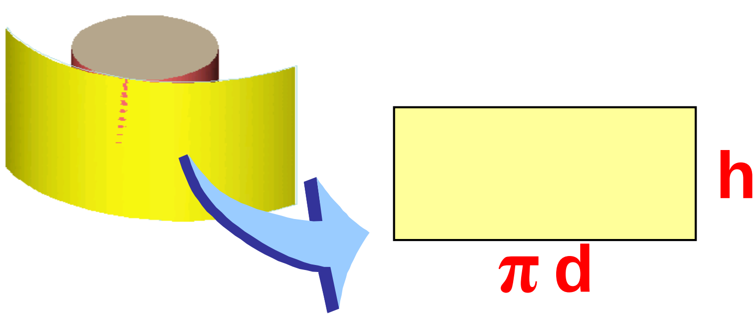
$$A = \pi r^2$$

$$V = \pi r^2 h$$

So the volume of a cylinder is calculated using this relationship (equation.)

The surface area of a cylinder is also easy to calculate. There are two circular end surfaces that are calculated using the same relationship (equation) that we just worked with:

$$A = \pi r^2$$



The area of the vertical, curved surface is easy to calculate if you visualize unwrapping it as though it were the paper label on a grocery can.

Since the unwrapped edge is the circumference of the circle, and the vertical edge is the height of the cylinder, the area can be represented by the following relation (equation.)

$$A = \pi d h$$

Total Surface Area

$$A=2\pi r^2+\pi dh$$

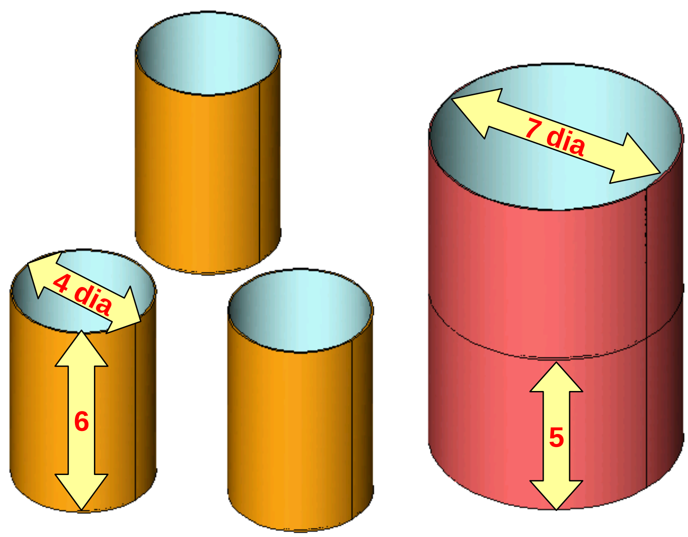
The total surface area is calculated by combining the two circular areas and the one unwrapped area using the following relationship (equation.)

Let’s solve a simple problem using the volume of a cylinder relationship.

Let’s suppose you have a large cylindrical container that is partially full of paint. You would like to pour the paint into three smaller containers that are all the same size but you are not sure if they can hold all of the paint. It would be nice to calculate an answer before messing with the paint.

You measure the small containers and find that each one is 4 inches in diameter and 6 inches high.

The large container is 7 inches in diameter and is filled to a depth of 5 inches with the paint.



Our first task is to determine how much paint is in the large container.

We know that the volume can be calculated using the following relation. (equation.)

$$V=\pi r^2h$$

Plugging in the numbers and using 3.14 for the value of π we get:

V= 3.14 x 3.5 x 3.5 x 5

V = 192.325 cubic inches

So this is the amount of paint that we have to pour into the three smaller containers.

Next, let's find the volume of one of the small containers.

We'll use the same relationship (equation.)

$$V = \pi r^2 h$$

Plugging in the numbers we get:

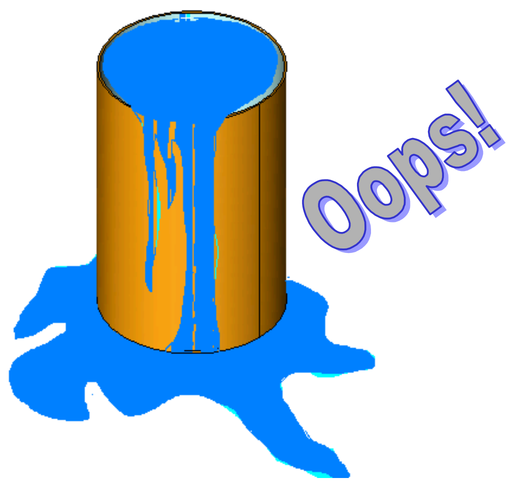
$$V = 3.14 \times 2 \times 2 \times 6$$

$$V = 75.36$$

We have three small containers, so the total volume they can handle is:

$$3 \times 75.36 = 226.08$$

Since this is greater than the volume of paint in the big container, we can go ahead and pour the paint!



You will run into many problems that require you to calculate volumes of objects. Remember the following pointers when you handle these problems:

First: Write down the dimensions of each object in the problem. Sometimes the problem has a twist. In the example above, I could have been sneaky and also given you the height of the large container. For that problem, the actual height is not important, since the can is partially filled!

Second: Carefully calculate the volume of each object in the problem.

Third: Write down a relationship that describes the problem. For instance, in the problem above, we knew that the total of the three small volumes had to be greater than the volume of paint in the large can.